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Idiosyncratic Remarks by a Bibliomaniac 7. Semipopular Mathematics

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There appears to be a general feeling in the science community that not enough popular exposition is done. Which also means that nearly every new effort is generally greeted with enthusiasm and praised without restraint. I beg to disagree. There is quite a lot and usually of good quality. On my own personal shelves, there are well over 200 popular science books, practically all in mathematics and physics and most of them of fairly recent vintage. Thus, there is room for being critical.

When popular exposition is done by first-rate scientists, such as Roger Penrose, Ian Stewart, Keith Devlin and V. I. Arnol'd, the results are generally excellent. When it is done by mediocre people or amateurs almost anything can happen: from truly excellent to deplorable and anywhere in between, as I will attempt to illustrate in this column.

Before doing so, let me state with some emphasis that I have admiration and more for those science publishers who (occasionally) publish popular science books (such as Wiley, Springer, and Oxford UP) and a sense of irritation with those that do not (such as Kluwer Academic Publishers and Elsevier Science Publishers).

One great thing about the late unlamented former Soviet Union was a tradition of popular exposition of the highest quality, often by the absolute top scientists, much of it still untranslated, and by now even unobtainable in the original form.

John L. Casti: *Five More Golden Rules*, Wiley, 2000, 268 pp., £12.50, ISBN 0-471-39528-5.

This is a successor volume to 'Five golden rules', also published by Wiley. For a review of that one see, e.g., the one by J. Appell in MATH (*Zentralblat der Mathematik und Grenzgebiete*).

The topics covered in this one are: The Alexander polynomial: knot theory; The Hopf bifurcation theorem: dynamical system theory; The Kalman filter: control theory; The Hahn–Banach theorem: functional analysis; The Shannon coding theorem: information theory.

This is a good selection, as were the five in the preceding volume; these are truly outstanding achievements of mathematics in the 20th Century and they have also been of enormous (though, publicly, basically unappreciated) importance as regards the appliances that make our current life a good deal more comfortable.

These topics are also not all that easy to write about for the intellectual layman. Casti does that well and he does not fall into the error of avoiding formulas. Indeed, if it be true, as has been said, that every formula in a book cuts its potential sales into half, than the sales of the present book cannot be expected to reach double digit figures. Casti knows and shows that formulas are more precise and enlightening than clumsy translations into everyday language which was never designed for the communication of ideas (but, instead, according to some authorities, for gossip, and, possibly, preaching).

Casti has read widely and he has much to say and does so with grace and charm. The examples and illustrations are well chosen and convincing. But he is sloppy, very sloppy, maybe hopelessly sloppy. This has been noted before, e.g., in the review by Appell already mentioned. Here are some examples from the present volume. There are many more. Figure 1.3 on page 8 and 1.5 on page 10 are, according to the captions, both about the figure-8 knot. They are very different and in fact figure 1.3 is not about the figure-8 knot at all. The statement of 'Alexander's polynomial invariant theorem' on page 22 is self-contradictory, confusing, and wrong. The 'Möbius band theorem' on page 27 is not understandable through the sudden appearance of edges named $b_i b_i'$ which do not appear anywhere else in the text or figures. The illustration on page 45 is a saddle point, which is not something that is usually called a point attractor. In fact, Casti happily and systematically mixes up the notion of an equilibrium of a dynamical system and that of an attractor. It is, of course, not true that a potential dynamical system can only have point equilibria (page 67). And, as in the precessor volume, he also messes up the notions of smooth and analytic.

Chapter 3 is about control theory. This is the area in which Casti has worked professionally. Even here, inaccuracies and mistakes crop up. As stated, the 'positive reachability theorem' on page 112 is simply wrong. There is such a theorem if one limits oneself to bounded controls and this idea is mentioned in the text. But it is far from clear that this restriction is to be included here.

All this is intensely irritating and one gets the feeling that one simply cannot trust the author's statements; not in mathematics, his own field, and probably still less in other areas of science. Many years ago, in connection with another controversy in which Casti was involved, I told our librarian that I would strangle him with my bare hands if he ever purchased a book by Casti; that injunction still holds.*

The publishers would do well in future to hire a really critical scientist to go through things before publishing further efforts by Casti. And that is probably

* For a whole host of other examples of Casti's sloppiness (the same word is in fact used there) and possibly even incompetence, see the review by G. M. Ziegler of 'Mathematical mountaintops' in 'MATH'. That book, apparently, has now been recalled by its publishers, Oxford University Press, loc. cit.

worth the extra work and money, because if the sloppiness had been removed, this would have been a very worthwhile book.

Benjamin H. Yandell: *The Honors Class*, A. K. Peters, 2002, 486 pp., £28.00, ISBN 1-56881-141-1, hbk.

This book is about Hilbert's 23 problems. It discusses the problems themselves, or at least does not shy away from them, but mostly it is about the people who tried to solve them or who were involved one way or another. And that is a very rich source of material for, by and large, mathematicians are most colourful and interesting people.

The author, Yandell, is an amateur in all the best meanings of the word. That is, he is not really a professional mathematician and he most obviously loves his subject. He has followed the excellent method of consulting the experts time and time again till he was absolutely sure that he had things right.* I expect that some of his victims in this got heartily tired of him. The result is worth the trouble. This is a fascinating book to read and it contains a wealth of anecdotal and biographic material that must have taken great effort to collect (though perhaps less than one might think because the favourite topic of conversation when mathematicians get together are the vagaries of, and stories about, other mathematicians).

All in all, I think this is an excellent effort, and much of the more rewarding material is in the semitechnical description of the real mathematics involved. Easy reading this book is definitely not; as the author recommends, read it like a scientist would; don't worry if something is for the moment not understandable; there is always a good chance that things will become clearer later, once your brain has had time to ruminate and dream a bit.

Usually there are occasional lapses in a book of this type, but I have not found any real ones. About the only thing I can cavil at is the impression given on pages 297ff that Paul Koebe really was (all that) famous; I, and others, think that fame basically existed only in his own mind.** There are a few typos. (A particularly charming one occurs on line −7 of page 81.)

No matter. This is a very rewarding book to buy and read, and those that have the courage and stamina to do so, will be richly compensated.

Donald C. Benson: *The Moment of Proof. Mathematical Epiphanies*, Oxford University Press, 2001, 331 pp., £9.99, ISBN 0-19-513919-4, Pb.

Aha, insight, eureka, the moment of truth, the joy of discovery, or even the prosaic q.e.d. There is nothing like it and all good scientists (and many more) have experienced it. The 'high' can be so intense that the scientist concerned for hours is

* For a rather concentrated survey of the current state of affairs of Hilbert's problems, see the article 'Hilbert problems' in M. Hazewinkel (ed.), *Encyclopaedia of Mathematics, Supplement II* (= volume 12), Kluwer Academic Publishers, 2000. It is nice to be able to remark that the census in the book under review, pages 385ff, by and large, agrees completely with the one of loc. cit.

** I would recommend the interested reader to read the relevant sections in the biography of L. E. J. Brouwer by D. van Dalen, Oxford University Press.

incapable of doing anything, not even verifying that his idea is correct; practically the only thing one can do is to take a long walk or some other vigorous exercise to try to calm down, and that can take days. A heroin or cocain or sexual high is nothing compared to it.

In this book, the author tries to share this intense joy with others who are not necessarily professional mathematicians or scientists. That is a rather different kind of popular science writing. It is nice to know that something is true; it is immeasurably more satisfying to understand why something is true.

The question arises, does the author succeed. On the whole, I would say, definitely yes. Benson sticks to things that are mathematically on the rather easy side but which do have that quality of unexpectedness and elegance that make the field like almost no other. And he is careful and precise in his explanations and extremely concerned with presenting things in such a way that anybody can follow his arguments. A great deal of effort and thought has gone into this aspect of the book.

Thus, we meet the games of Nim and Kayles, continued fractions, (linear) Diophantine equations, the Lloyd fifteen puzzle, the relation between double angle and right angle triangles, the relation between Mersenne primes and perfect numbers, various forms of Russell's barber paradox, cutting and pasting of figures in the plane, the prisoner's dilemma, linear programming, the Petersburg paradox, Russian peasant multiplication, the seven bridges of Königsberg, the chaos game (Sierpinski triangle), the Cantor set, and much much more.

This is an absolutely delightful book and one wonders whether, and fervently hopes that, the author will find time in his retirement to compose an equally careful volume concentrating on somewhat more advanced bits and pieces of mathematics.